Elsevier Editorial System(tm) for Applied Mathematics and Computation

Manuscript Draft

Manuscript Number: AMC-D-07-02659

Title: Generalized FLT

Article Type: Short Communication

Keywords: FTT,Primes,Co primes

Corresponding Author: Mr hemant ajay pandey,

Corresponding Author's Institution: City Montessary school

First Author: hemant ajay pandey

Order of Authors: hemant ajay pandey

Abstract: The Abstract: The generalized Fermat's last theorem is a open problem in number theory and it asks whether the equation

xa + yb = zc has a solution x, y, z, I + & a, b, c, 3 and x, y, z are mutually co-prime.

As we all are well aware of the celebrated proof of Fermat's last theorem by legendry Sir Andrew wiles in 1974.The truth of FLT in general case when all the exponents are different integers larger than 3, is the main point of discussion of present paper.

We have attempted to establish the truth of the result through the method of contradiction.

Our method from the start assumes that x,y,z, are mutually co prime and in the progress of our solution we try to establish that for a solution they can't be co prime.

Title Page

1. ***Contents***
2. ***Author s Profile***
3. ***Cover Letter***
4. ***Abstract***
5. ***Manuscript***
6. ***References***

AUTHORS PROFILE

Author: Hemant Pandey

AffiliationPresently working as an IIT JEE lecturer for TIME institute:

City Montessary School (CMS) –Innovation Wing (IW),Amelox college tutor
www.amelox.com

Job Profile: Content Developer; Writer,IIT JEE lecturer

Branch: Main Branch,

Station Road, Lucknow

Writing profile: Currently working on a text book for

Grade 11-12 of mathematics with Amelox College Tutor; California (USA)

Qualification: Masters of Science in Mathematics and currently a Researcher

Postal Address:

16/732, Indira Nagar,Lucknow.
Uttar Pradesh – 226016(Pin) India
Mobile Number +91-9336291008, +91-9338202560

Mailing Address:

E-mail- *ajay_fermat@rediffmail.com*

E-mail- ajay_euler@rediffmail.com
E-mail- ajay_gauss@rediffmail.com

Cover Letter

Dear Sir,

My name is hemant pandey and I am a young researcher.

I am proposing an paper on generalized FLT for possible publication. The main argument used in the paper is the method of contradiction and property of all mutually co prime numbers, that each number in the universe is expressible as a linear combination of two co prime numbers.

This is to certify that the work is original and has not been submitted any where else for publication consideration.

Details are in the pdf attachment.

Sincerely,

Hemant Pandey

A Generalization to FLT

The Abstract: *The generalized Fermat's last theorem is a open problem in number theory and it asks whether the equation*

$x^a + y^b = z^c$ has a solution $\forall$ $x, y, z, \in I^+$ & $a, b, c, \geq 3$ and x, y, z are mutually co-prime.

As we all are well aware of the celebrated proof of Fermat's last theorem by legendry Sir Andrew wiles in 1974.The truth of FLT in general case when all the exponents are different integers larger than 3, is the main point of discussion of present paper.

We have attempted to establish the truth of the result through the method of contradiction.

Our method from the start assumes that x,y,z, are mutually co prime and in the progress of our solution we try to establish that for a solution they can't be co prime.

Meaning of symbols:

x^k	-	*x raised to the power 'k'*
$\forall$	-	*For all*
I^+	-	*set of + ve integers*
$\Rightarrow$	-	*Implies that*
$>$	-	*Is greater than*
$<$	-	*Is less than*
$=$	-	*Is equal to*
$\neq$	-	*Is not equal to*
$\in$	-	*Belongs to*

The Manuscript

1. Introduction

The generalization of FLT can be expressed in a classic one line statement:

The following equation has no solution, for all x, y, z $\in$ I^+ *and mutually co-prime.*

a ,b , c are integers greater than 2.

$\therefore \qquad x^a + y^b \# z^c \qquad .....[A]$

If x ,y , z are co prime, $\in$ I^+

And a,b,c are integers greater than or equal to three.

The following proposed proof works on the method of contradiction.

2.Proof:

We would start our proof with a known result that for all x, y , z mutually co-prime we can find l, m, mutually co-prime

s.t $\quad lx-my = 1 \qquad$ *note that l & y and x & m are coprime.*

$\Rightarrow \quad z = (l \,.\, z) \,.\, x - (m \,.z\,) \,.y$

$\forall \ z \in \ I^+$

$\Rightarrow$ *We can always find A ,B* $\forall$ *x ,y , z ,where the three variables are co prime.*

s.t $z = Ax - By$ *,where* $A=l.z$ *and* $B=m.z$

Rewriting (A) we get

$x^a + y^b \ \# \ (Ax - By)^C$

is equivalent generalized Fermat's last theorem.

$\therefore$ *To employ the method of contradiction assuming the contrary*

Let $x^a + y^b = (Ax - By)^c$ *for some* $\forall$ *x,y co prime* ..... (B)

Note back since $A = l.z$

$B = m.z$

A , y & B , x , are mutually co prime as y & z and x & z are mutually co prime.
...... (R-1)

Now (B) gives

$x^a + y^b = (A.x)^c - (A.x)^{c-1}.{}^cC_1.(By) + \dots\dots\dots + (-1)^C.(By)^C$

Rearranging we get $x^a - (A.x)^c = -y^b - (Ax)^{C-1}.{}^cC_1.(By) + \dots\dots + (-1)^C.(By)^C$

$X^a[1 - A^c(x)^{c-a}] = -y\{y^{b-1} + (A.x)^{c-1}.{}^cC_1.B + \dots\dots + (-1)^{c-1}.(By)^{c-1}\}$

or $x^a[1 - A^c(x)^{c-a}] = -y\{\ y^{b-1} + (A.x)^{c-1}.{}^cC_1.B - \dots\dots + (-1)^{c-1}.(B^c.y^{c-1}\}$ ....(C)

Since x & y co-prime

$\Rightarrow 1 - (A.x)^{c-a} = \lambda_1 y$ [where λ_1 = integer] ....(1)

Now putting this value in (C) we have

$x^a \lambda_1 y = -y\{\ y^{b-1} + (A.x^{c-1}.{}^cC_1.B - \dots\dots + (-1)^{c-1}.(B^c.y^{c-1}\}$

Or $x^a \lambda_1 = y^{b-1} + (A.x)^{c-1}.{}^cC_1.B - \dots. + (-1)^{c-1}.B^c.y^{c-1}$

$\Rightarrow x^a \lambda_1 = y^{b-1} + x[A^{c-1}.x^{c-2}.{}^cC_1.B - \dots\dots\dots + (-1)^{c-2}.B^{c-1}.y^{c-2}A] + (-1)^{c-1}.B^c..y^{c-1}$

Or $x^a \lambda_1 = y^{b-1} + (-1)^{c-1}.B^c.y^{c-1} + x[A^{c-1}.x^{c-2} - {}^cC_1.B + \dots\dots\dots + (-1)^{c-2}.B^{c-1}.y^{c-2}.A]$

Or $x^a \lambda_1 = y^{b-1}[\, 1 + (-1)^{c-1}.B^c.y^{\,c-b}] + x[\, A^{\,c-1}.x^{\,c-2} - {}^cC_1.B + ... + (-1)^{c-2}.B^{\,c-1}.{}^2.A]$

$\Rightarrow \;\; y^{\,b-1} + (-1)^{c-1}.B^c.y^{\,c-1} = \lambda_2 x$

Since other summand is a multiple of x.

Or $y^{\,b-1}\{1 + (-1)^{c-1\cdot}B^c.y^{\,c-b}\} = \lambda_2 x$

Now x & y are co prime

$\Rightarrow \;\; 1 + (-1)^{c-1\cdot}B^c.y^{\,c-b} \quad = \lambda_2 x......(2)$

from 1 & 2 we have

$1 - (A.)^c x^{c-a} = \lambda_1 y$ & $1 + (-1)^{c-1}.B^c.y^{\,c-b} = \lambda_2\, x \qquad ....(3)$

Here $\lambda_{1\&}\ \lambda_2$ *are arbitrary integers to be determined.*

Note from above situation a # b # c

if a =b =c the conditions cannot hold good, as the variables become unity.

This excludes the cases of FLT to be encountered here.

a # b # c is also a condition.

Here we have tacitly assumed that c > a > b but that does not affect generality.

As if a > b > c we may take

$x^a = z^c - y^b$ *etc.*

all the other steps are similar

The greatest exponent is to be equated to the sum or difference of other two.

Now set (3) gives that

$x(A)^c.x^{c-a-1} + \lambda_1 y = 1$

& $\lambda_2 .x + y(B^c.y^{c-b-1})(-1)^c = 1$

A little algebra gives that for a solution

$$\frac{A^{c-a}.x^{c-a-1}}{\lambda_2}=\frac{\lambda_1}{B^c.y^{c-b}.(-1)^c}=1$$

$\lambda_2 = A^c.x^{c-a-1}$

$\lambda_1 = B^c.y^{c-b-1}(-1)^c$

Putting the above values in the equation (C) we get

$x^a.B^c.y^{c-b}(-1)^c = -y^b[1+(-1)^{c-1}.B^c.y^{c-b-1}] - y[(A.x)^{c-1}.{}^cC_1.B......+(-1)^{c-2}B^{c-1}.y^{c-2}.x.A\}]$

Or $x^a.B^c.y^{c-b}(-1)^c = (-1)\{y^b.A^c.x^{c-a-1}.x\} - y[(Ax)^{c-1}.{}^cC_1.B+........+(-1)^{c-2}.B^{c-1}.y^{c-2}.x.A\}]$

Or $x^a.B^c.y^{c-b}.(-1)^c + \{y^b.A^c.x^{c-a}\} = -y[(Ax)^{c-1}.{}^cC_1.B+........+(-1)^{c-2}.B^{c-1}.y^{c-2}.x.A\}]$

Or $x^a.B^c.y^{c-b}.(-1)^c + \{y^b.A^c.x^{c-a}\} + (A.x)^c + (-1)^C(By)^C$

$= -y[(Ax)^{c-1}.{}^cC_1.B+........+(-1)^{c-2}.B^{c-1}.y^{c-2}.x.A\}] + (A.x)^c + (-1)^C(By)^C$

$\Rightarrow x^a.[B^c.y^{c-b}.(-1)^c + .A^c.x^{c-a}] + y^b.[A^c.x^{c-a} + B^c.y^{c-b}.(-1)^c] = (Ax - By)^C$

Comparing with (B) we have

$B^c.y^{c-b}.(-1)^c + .A^c.x^{c-a} = 1$

Which is certainly impossible, since B=l.z & A=m.z

Only possibility may be z=1, But that case is taken care of via catalan's conjecture's proof.

We may formally prove the case even.

$Z=1 \Rightarrow m^c.y^{c-b}.(-1)^c + .l^c.x^{c-a} = 1$

But l.x-m.y=1

$\Rightarrow l^c.x^{c-a-1}/l = m^c.y^{c-b-1}.(-1)^c/-m$

$\Rightarrow l^{c-1}.x^{c-a-1} = m^{c-1}.y^{c-b-1}.(-1)^{c-1}$

Which is certainly untrue since l , y & m are co prime etc.

Hence $x^a + y^b = z^c$

has no soln. $\forall$

a # b # c

& x,y,z co prime.

Hence the result.

Sole Author

Hemant Pandey

Title Page

AUTHORS PROFILE

Author: Hemant Pandey

Affiliation Presently working as an IIT JEE lecturer for TIME institute:

City Montessary School (CMS) –Innovation Wing (IW),Amelox college tutor **www.amelox.com**

Job Profile: Content Developer; Writer,IIT JEE lecturer

Branch: Main Branch,

Station Road, Lucknow

Writing profile: Currently working on a text book for

Grade 11-12 of mathematics with Amelox College Tutor; California (USA)

Qualification: Masters of Science in Mathematics and currently a Researcher

Postal Address:

16/732, Indira Nagar,Lucknow.
Uttar Pradesh – 226016(Pin) India
Mobile Number +91-9336291008, +91-9338202560

Mailing Address:

E-mail- *ajay_fermat@rediffmail.com*

E-mail- ajay_euler@rediffmail.com
E-mail- ajay_gauss@rediffmail.com

Cover Letter

Dear Sir,

My name is hemant pandey and I am a young researcher.

I am proposing an paper on generalized FLT for possible publication. The main argument used in the paper is the method of contradiction and property of all mutually co prime numbers, that each number in the universe is expressible as a linear combination of two co prime numbers.

This is to certify that the work is original and has not been submitted any where else for publication consideration.

Details are in the pdf attachment.

Sincerely,

Hemant Pandey

A Generalization to FLT

The Abstract: *The generalized Fermat's last theorem is a open problem in number theory and it asks whether the equation*

$x^a + y^b = z^c$ has a solution $\forall$ $x, y, z, \in I^+$ & $a, b, c, \geq 3$ and x, y, z are mutually co-prime.

As we all are well aware of the celebrated proof of Fermat's last theorem by legendry Sir Andrew wiles in 1974.The truth of FLT in general case when all the exponents are different integers larger than 3, is the main point of discussion of present paper.

We have attempted to establish the truth of the result through the method of contradiction.

Our method from the start assumes that x,y,z, are mutually co prime and in the progress of our solution we try to establish that for a solution they can't be co prime.

Meaning of symbols:

x^k	-	*x raised to the power 'k'*
$\forall$	-	*For all*
I^+	-	*set of + ve integers*
$\Rightarrow$	-	*Implies that*
$>$	-	*Is greater than*
$<$	-	*Is less than*
$=$	-	*Is equal to*
$\neq$	-	*Is not equal to*
$\in$	-	*Belongs to*

The Manuscript

1. Introduction

The generalization of FLT can be expressed in a classic one line statement:

The following equation has no solution, for all x, y, z $\in I^{+}$ *and mutually co-prime.*

a ,b , c are integers greater than 2.

$\therefore \quad x^a + y^b \# z^c \qquad .....[A]$

If x ,y , z are co prime, $\in \; I^{+}$

And a,b,c are integers greater than or equal to three.

The following proposed proof works on the method of contradiction.

2.Proof:

We would start our proof with a known result that for all x, y , z mutually co-prime we can find l, m, mutually co-prime

s.t $lx - my = 1$ *note that l & y and x & m are coprime.*

$\Rightarrow \quad z = (l \cdot z) \cdot x - (m \cdot z) \cdot y$

$\forall\ z \in\ I^+$

$\Rightarrow$ *We can always find A ,B* $\forall$ *x ,y , z ,where the three variables are co prime.*

s.t $z = Ax - By$ *,where* $A=l.z$ *and* $B=m.z$

Rewriting (A) we get

$x^a + y^b \# (Ax - By)^C$

is equivalent generalized Fermat's last theorem.

$\therefore$ *To employ the method of contradiction assuming the contrary*

Let $x^a + y^b = (Ax - By)^c$ *for some* $\forall$ *x,y co prime* ..... (B)

Note back since $A = l.z$

$B = m.z$

A , y & B , x , are mutually co prime as y & z and x & z are mutually co prime.
...... (R-1)

Now (B) gives

$x^a + y^b = (A.x)^c - (A.x)^{c-1}.{}^cC_1.(By) + \ldots\ldots\ldots + (-1)^C.(By)^C$

Rearranging we get $x^a - (A.x)^c = -y^b - (Ax)^{C-1}.{}^cC_1.(By) + \ldots\ldots + (-1)^C.(By)^C$

$X^a[1 - A^c(x)^{c-a}] = -y\{y^{b-1} + (A.x)^{c-1}.{}^cC_1.B + \ldots\ldots + (-1)^{c-1}.(By)^{c-1}\}$

or $x^a[1 - A^c(x)^{c-a}] = -y\{\ y^{b-1} + (A.x)^{c-1}.{}^cC_1.B - \ldots\ldots + (-1)^{c-1}.(B^c.y^{c-1}\}$ ....(C)

Since x & y co-prime

$\Rightarrow 1 - (A.x)^{c-a} = \lambda_1 y$ [where λ_1 = integer] ....(1)

Now putting this value in (C) we have

$x^a\lambda_1 y = -y\{\ y^{b-1} + (A.x^{c-1}.{}^cC_1.B - \ldots\ldots + (-1)^{c-1}.(B^c.y^{c-1}\}$

Or $x^a\lambda_1 = y^{b-1} + (A.x)^{c-1}.{}^cC_1.B - \ldots. + (-1)^{c-1}.B^c.y^{c-1}$

$\Rightarrow$ $x^a\lambda_1 = y^{b-1} + x[A^{c-1}.x^{c-2}.{}^cC_1.B - \ldots\ldots\ldots + (-1)^{c-2}.B^{c-1}.y^{c-2}A] + (-1)^{c-1}.B^c..y^{c-1}$

Or $x^a\lambda_1 = y^{b-1} + (-1)^{c-1}.B^c.y^{c-1} + x[A^{c-1}.x^{c-2} - {}^cC_1.B + \ldots\ldots.. + (-1)^{c-2}.B^{c-1}.y^{c-2}.A]$

Or $x^a \lambda_1 = y^{b-1}[\ 1 + (-1)^{c-1}.B^c.y^{c-b}] + x[\ A^{c-1}.x^{c-2} - {}^cC_1.B + ...+(-1)^{c-2}.B^{c-1}.^2.A]$

$\Rightarrow \quad y^{b-1} + (-1)^{c-1}.B^c.y^{c-1} = \lambda_2 x$

Since other summand is a multiple of x.

Or $y^{b-1}\{1 + (-1)^{c-1}.B^c.y^{c-b}\} = \lambda_2 x$

Now x & y are co prime

$\Rightarrow \quad 1 + (-1)^{c-1}.B^c.y^{c-b} \qquad = \lambda_2 x......(2)$

from 1 & 2 we have

$1 - (A.)^c x^{c-a} = \lambda_1 y$ & $1 + (-1)^{c-1}.B^c.y^{c-b} = \lambda_2 x \qquad ....(3)$

Here λ_1 & λ_2 are arbitrary integers to be determined.

Note from above situation a # b # c

if a =b =c the conditions cannot hold good, as the variables become unity.

This excludes the cases of FLT to be encountered here.

a # b # c is also a condition.

Here we have tacitly assumed that c > a > b but that does not affect generality.

As if a > b > c we may take

$x^a = z^c - y^b$ etc.

all the other steps are similar

The greatest exponent is to be equated to the sum or difference of other two.

Now set (3) gives that

$x(A)^c.x^{c-a-1} + \lambda_1 y = 1$

& $\lambda_2.x + y(B^c.y^{c-b-1})(-1)^c = 1$

A little algebra gives that for a solution

$$\frac{A^{c-a}.x^{c-a-1}}{\lambda_2}=\frac{\lambda_1}{B^c.y^{c-b}.(-1)^c}=1$$

$\lambda_2 = A^c.x^{c-a-1}$

$\lambda_1 = B^c.y^{c-b-1}(-1)^c$

Putting the above values in the equation (C) we get

$x^a.B^c.y^{c-b}(-1)^c = -y^b [1 + (-1)^{c-1}.B^c.y^{c-b-1}] - y[(A.x)^{c-1}.\,{}^cC_1.B......+(-1)^{c-2} B^{c-1}.y^{c-2}.x.A\}]$

Or $x^a.B^c.y^{c-b}(-1)^c = (-1)\{y^b.A^c.x^{c-a-1}.x\} - y[(Ax)^{c-1}.{}^cC_1.B+........+(-1)^{c-2}.B^{c-1}.y^{c-2}.x.A\}]$

Or $x^a.B^c.y^{c-b}.(-1)^c + \{y^b.A^c.x^{c-a}\} = -y[(Ax)^{c-1}.{}^cC_1.B+........+(-1)^{c-2}.B^{c-1}.y^{c-2}.x.A\}]$

Or $x^a.B^c.y^{c-b}.(-1)^c + \{y^b.A^c.x^{c-a}\} + (A.x)^c + (-1)^C(By)^C$

$= -y[(Ax)^{c-1}.{}^cC_1.B+........+(-1)^{c-2}.B^{c-1}.y^{c-2}.x.A\}] + (A.x)^c + (-1)^C(By)^C$

$\Rightarrow x^a.[B^c.y^{c-b}.(-1)^c + .A^c.x^{c-a}] + y^b.[A^c.x^{c-a} + B^c.y^{c-b}.(-1)^c] = (Ax - By)^C$

Comparing with (B) we have

$B^c.y^{c-b}.(-1)^c + .A^c.x^{c-a} = 1$

Which is certainly impossible, since B=l.z & A=m.z

Only possibility may be z=1, But that case is taken care of via catalan's conjecture's proof.

We may formally prove the case even.

$Z=1 \Rightarrow m^c.y^{c-b}.(-1)^c + .l^c.x^{c-a} = 1$

But $l.x-m.y=1$

$\Rightarrow l^c.x^{c-a-1}/l = m^c.y^{c-b-1}.(-1)^c/-m$

$\Rightarrow l^{c-1}.x^{c-a-1} = m^{c-1}.y^{c-b-1}.(-1)^{c-1}$

Which is certainly untrue since l , y & m are co prime etc.

Hence $x^a + y^b = z^c$

has no soln. $\forall$

a # b # c

& x,y,z co prime.

Hence the result.

Sole Author

Hemant Pandey